Ambient Sea Noise Directionality: Measurement and Processing

Nai-chyuan Yen
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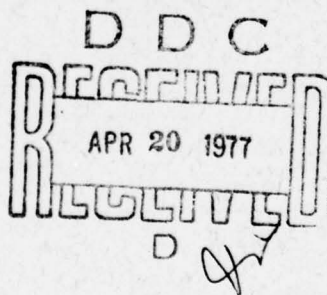
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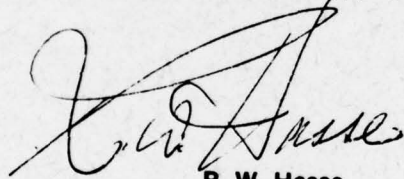Newport, Rhode Island ● New London, Connecticut

PREFACE

This report was prepared under NUSC Project No.
A-650-05, "Ambient Noise Characteristics Affecting
Sonar and Physical Oceanography," Navy Subproject No.
SF 52-552-602, Principal Investigator, R. L. Martin,
Code 312. The sponsoring activity was the Naval Sea
Systems Command, Program Manager, A. Franceschetti,
SEA-06H1-4.

The Technical Reviewer for this report was Dr.
A. H. Nuttall, Code 313.

The author gratefully acknowledges the many
stimulating discussions he had with R. L. Martin and
R. J. Nielsen.

REVIEWED AND APPROVED: 28 February 1977

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Laboratory, Naval Underwater Systems Center,
New London, Connecticut 06320.

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NUSC-TR-5545	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) AMBIENT SEA NOISE DIRECTIONALITY: MEASUREMENT AND PROCESSING.		5. TYPE OF REPORT & PERIOD COVERED Technical report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Nai-chyuan Yen		8. CONTRACT OR GRANT NUMBER(s) F52552 SF52552602
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Underwater Systems Center New London Laboratory New London, CT 06320		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS A-650-05 SF 52 552-602
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE 28 February 1977
		13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Acoustic Array Directionality	Coherent Noise Sources
Ambient Sea Noise Directionality	Cross-Spectral Matrix
Angular Resolution	Directional AUTOBUOY

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The fundamental relationships between acoustic array parameters and the noise field are reviewed to establish the connection among the various processing techniques for estimation of the ambient-sea-noise directionality. The analyses, based on the concept of multidimensional space, conclude that basic information about the ambient-sea-noise directionality can be obtained directly from the cross-spectral matrix of the array's hydrophone outputs. The angular resolution of the processed noise field depends on the array structure, the stationarity of the noise field, and the precision in numerical computation. The cross-spectral

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20. Abstract (Cont'd)

data can also be used to indicate the dominant coherent noise sources in a certain multidimensional representation. Examples of a simulated noise field and actual sea data are illustrated for the directional AUTOBUOY.

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INTRODUCTION

In the study of ambient noise in the ocean, the spatial coherence of the noise field can be characterized by the angular distribution and strengths of the noise arrivals. Such a description of the noise field is generally very useful in analyzing the causes of ambient sea noise and its relation to the ocean environment. Furthermore, ambient-sea-noise directionality data provide the basic information in designing the receiving array used in underwater communication systems.

A considerable amount of work concerning ambient-sea-noise directionality has been conducted since the early part of 1960.[1,2,3] As noise directionality is not a quantity which can be measured directly, many different processing methods have been employed to extract ambient-noise-directionality data from the output of an array of receiving hydrophones. The theoretical analysis of the relationship between the array output and noise field[4,5,6] indicates that the exact noise field (unique solution for the mathematical problem) is difficult to determine, and only limited information about the noise field (principal solution) can be recovered by an array with a finite aperture size.

Techniques developed to estimate the noise field are based on either the array beamformer output[2,3,7] or the cross-spectral matrix between its hydrophone outputs.[8] Because of the complications in deriving the processing algorithm, discussions of those techniques are generally confined to an equally spaced hydrophone line array. Utilization of a deconvolution technique for nonuniformly spaced arrays has been reported recently in the literature.[9] The analytical properties of such a specific algorithm have not been elaborated on yet.

On a separate front, research efforts have been directed to the improvement of array design, the measurement tool for ambient-sea-noise directionality. Various approaches, some of which are extensions of knowledge accumulated in fields such as radio astronomy and geophysics, have been devised to optimize array performance so that the array output can be related directly to the noise field. Those approaches usually involve computation of a set of shading factors, amplitude and phase, applied to array hydrophones to maximize certain array performance parameters: directivity index,[10,11] main-lobe to side-lobe ratio,[12,13] or high-order pressure gradient.[14]

Depending on which array parameter has been optimized, the array beamformer output will have least interference from its side-lobe response; hence, its measurement can be more closely related to the noise field in the direction in which the array is steered. Since the noise field is not known beforehand, the optimization of array performance has to be based on an assumed noise field, generally isotropic, which is not necessarily true in reality. Such a shortcoming is eliminated only after the optimum-adaptive beamforming technique[15] is adopted. However, the processing system for carrying out this technique is complicated.[16] In addition, even if the array gain is optimized with this processing method, the width of the array's main lobe varies with steering direction and depends on the noise field it intends to measure. Thus, further interpretation or processing is needed to express the measurement results in units of noise level per steradian.

This report presents the results of an investigation of measurement and processing methods for ambient-sea-noise directionality used in a receiving array. The purpose of such a study is to emphasize the connections among the various processing techniques and to establish a common reference for noise-directionality data. In order to reconcile the theoretical analysis and experimental measurement of ambient-sea-noise directionality, the basic relationships between array parameters and the noise field are reexamined. Discussions are directed also to the practical problems occurring in the real ocean environment, such as stationarity and homogeneity of the noise field.

Derivation of ambient-sea-noise directionality in terms of the measurable quantities of the receiving array are followed through in great detail in this report. In this way, a proper physical interpretation can be applied to the final processed ambient-sea-noise directionality data. The limitation of the numerical computation in carrying out data reduction has been addressed with consideration for availability of existing computer algorithms. Although the overall analysis presented in this report can be directed to any type of array used for ambient-sea-noise directionality measurement, only a vertical AUTOBUOY directional array[17] is applied as an example in the computation of data reduction. This is due mainly to the availability of field data for the AUTOBUOY,[18] so that comparisons can be made between a simulated noise field and a real noise field obtained from an actual ocean environment. It is hoped that this report can bridge some of the gaps in the interpretation of measurement results and the understanding of the noise field.

NOISE FIELD AND ARRAY PARAMETERS

FUNDAMENTAL RELATIONSHIPS

Spatial Correlation of Noise Field

A plane wave of amplitude n and angular frequency ω can be expressed mathematically as

$$ne^{-j(\omega t - \bar{k}\cdot\bar{r})} , \tag{1}$$

where $\bar{k}$ is the propagation vector, with a magnitude of $2\pi/\lambda$, and $\bar{r}$ is a direction vector with reference to a particular set of coordinates.

A receiving hydrophone placed at $\bar{s}_\ell$ from the origin will have response

$$e_\ell = ne^{j\bar{k}\cdot\bar{s}_\ell} , \tag{2}$$

where unit gain has been assumed for the hydrophone sensitivity and the time-dependent factor $e^{-j\omega t}$ is dropped for simplicity. The expression for e_ℓ in equation (2) can be considered as a vector (rotating vector or phasor) whose magnitude, n, and phase angle, $\bar{k}\cdot\bar{s}_\ell$, contain the information on the propagation direction of this plane wave.

Since the reference of the phase angle is chosen arbitrarily, measurements of this plane wave at two locations at least are needed to retain the phase angle information. As shown in figure 1, for two hydrophones located at $\bar{s}_\ell$ and $\bar{s}_m$, the phase angle information can be recovered simply by taking the cross product of the hydrophone outputs:

$$e_\ell e_m{}^* = n^2 e^{j\bar{k}\cdot(\bar{s}_\ell - \bar{s}_m)}$$

$$= n^2 e^{j\bar{k}\cdot\bar{d}_{\ell m}} , \tag{3}$$

where * denotes complex conjugation.

When the noise field is the result of signals arriving from various angular directions, both n and $\bar{k}$ will be functions of angle Ω, and the total response for the cross-product of hydrophone outputs will be

$$e_\ell e_m{}^* = \int n^2(\Omega) e^{j\bar{k}(\Omega)\cdot\bar{d}_{\ell m}} d\Omega . \tag{4}$$

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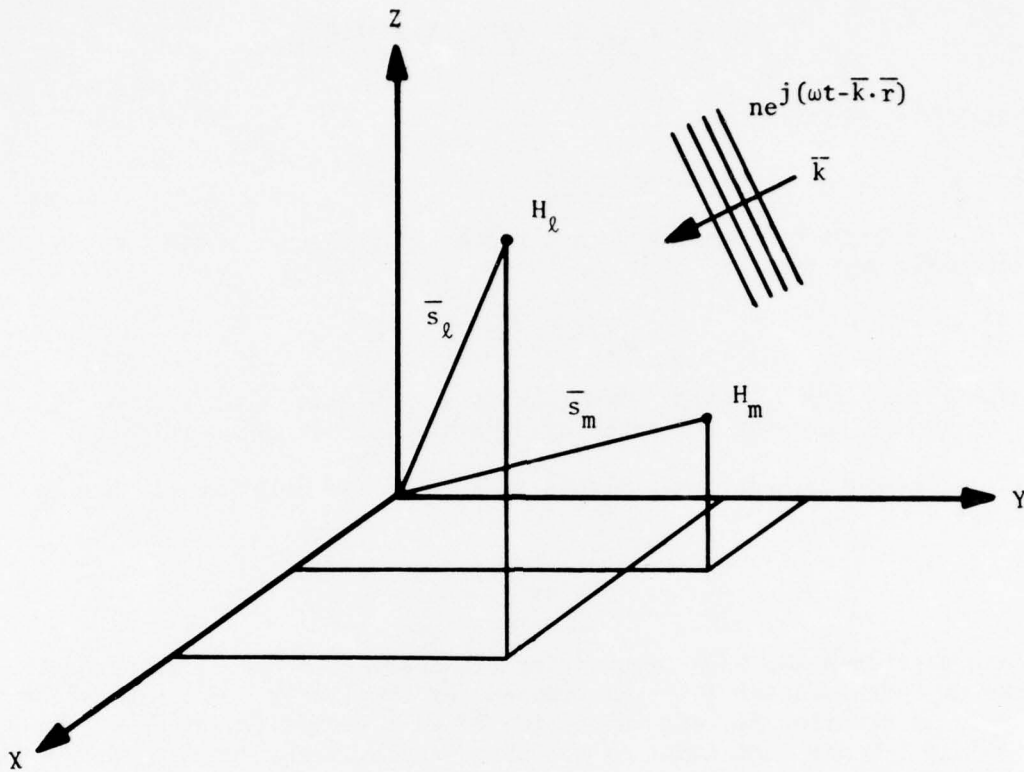
Figure 1. Measurement of Noise Field

Since equation (4) concerns only a single frequency ω (narrowband signal), it can be written more explicitly as

$$\gamma(\omega)_{\ell m} = \int N(\Omega,\omega) e^{j\bar{k}(\Omega,\omega)\cdot\bar{d}_{\ell m}} d\Omega \ , \qquad (5)$$

where $\gamma(\omega)_{\ell m}$ is the cross spectrum of hydrophone outputs ℓ and m separated by $\bar{d}_{\ell m}$, and $N(\Omega,\omega)$ is the power spectrum of the noise field arriving from direction Ω.

For noise directionality measurements, an array of several hydrophones will be employed. Thus different values of $\gamma_{\ell m}$ will be obtained, depending upon the hydrophone spacing in the array. The cross spectrum

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$\gamma_{\ell m}$ can be related to the noise field as expressed in equation (5). Nuttall[8] has pointed out that the noise field can be estimated directly from the cross-spectral matrix $\{\gamma_{\ell m}\}$ without further array processing, and he gave a detailed discussion. Derivation of those relationships will be examined again here, but with a different approach. This is not only to verify the previous result but also to establish a new concept for array processing.

For an array of M hydrophones, immersed in a stationary, homogeneous noise field, there will be a maximum of $L = 1 + \frac{M(M-1)}{2}$ independent measurements of $\gamma_{\ell m}$, if only a single frequency is considered (the independent measurements will be reduced to M for an equally-spaced hydrophone array). Consequently, there will be L sets of simultaneous equations of the type shown in equation (5). Using matrix notation (labeled with underline, number of lines will designate the dimension of the matrix), we find

$$
\underline{R} = \begin{vmatrix} \gamma_{1,1} \\ \gamma_{1,2} \\ \cdot \\ \cdot \\ \gamma_{\ell,m} \\ \cdot \\ \cdot \\ \gamma_{(M-1),M} \end{vmatrix} \qquad \underline{D} = \begin{vmatrix} e^{j\bar{k}\cdot\bar{d}_{1,1}} \\ e^{j\bar{k}\cdot\bar{d}_{1,2}} \\ \cdot \\ \cdot \\ e^{j\bar{k}\cdot\bar{d}_{\ell_1 m}} \\ \cdot \\ \cdot \\ e^{j\bar{k}\cdot\bar{d}_{(M-1),M}} \end{vmatrix} . \tag{6}
$$

Equation (5) can be expressed as:

$$
\underline{R} = \int N\underline{D}\, d\Omega . \tag{7}
$$

Recalling that $\underline{R}$ can be obtained from the cross products of array outputs and that $\underline{D}$ can be determined according to the array's structure, only N is unknown in equation (7). Basically, an integral equation in the form of equation (7) (Fredholm[19] equation of the first kind) will have an infinite set of solutions for unknown N. However, because there are available a finite set of independent measurements with the array, a unique solution for N can be deduced from information about $\underline{R}$ and $\underline{D}$. The transformation method will be used to derive the following relations.

Consider $\underline{D}$ to be a function whose bases are $e^{j\bar{k}\cdot\bar{d}_{\ell_1 m}}$. Apply Schmidt's orthonormalization process[20] to $\underline{D}$ and express $\underline{D}$ in new bases as

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$$\underline{D}' = \underline{\underline{Q}}\ \underline{D} = \begin{vmatrix} a_{11} & 0 & 0 & \cdots & 0 \\ a_{21} & a_{22} & 0 & & 0 \\ a_{31} & a_{32} & a_{33} & & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ a_{L1} & a_{L2} & a_{L3} & \cdots & a_{LL} \end{vmatrix} \begin{vmatrix} e^{j\bar{k}\cdot\bar{d}_1} \\ e^{j\bar{k}\cdot\bar{d}_2} \\ e^{j\bar{k}\cdot\bar{d}_3} \\ \cdot \\ \cdot \\ \cdot \\ e^{j\bar{k}\cdot\bar{d}_L} \end{vmatrix}$$

(8)

$$= \begin{vmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_L \end{vmatrix},$$

where the elements of $\underline{\underline{Q}}$ are determined by

$$\int u_i u_j {}^*d\Omega = 0 \text{ for } i \neq j$$

$$= 1 \text{ for } i = j\ ,$$

(9)

or, $\int \underline{D}'\underline{D}'^{\dagger}d\Omega = \underline{\underline{I}}$,

and the double subscript ℓm is replaced by k with the relation:

$$k = \ell + m - 1\ .$$

(10)

Since the transformation matrix $\underline{\underline{Q}}$ is a constant matrix (all its elements are constants determined in equation (9)), it can operate on both sides of equation (7); the result is

6

$$\underline{R}' = \underline{\underline{Q}}\ \underline{R} = \begin{vmatrix} \gamma_1' \\ \gamma_2' \\ \cdot \\ \cdot \\ \cdot \\ \gamma_k' \\ \cdot \\ \cdot \\ \cdot \\ \gamma_L' \end{vmatrix}$$

(11)

$$= \int N\underline{\underline{Q}}\ \underline{D}d\Omega = \int N\underline{D}'d\Omega = \int N \begin{vmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ \cdot \\ u_k \\ \cdot \\ \cdot \\ \cdot \\ u_L \end{vmatrix} d\Omega \ .$$

Because of the new bases, u's are orthonormal to each other and related to the element spacing of the array. We can assume $N = (\underline{D}')\dagger\underline{c}$ ($\underline{c}$ is a constant matrix (i.e., independent of Ω) to be determined). To obtain the unique solution based on array structure, equation (11) gives the following identity:

$$\begin{aligned} \underline{R}' &= \int N\underline{D}'d\Omega = \int \underline{D}'Nd\Omega \\ &= \int \underline{D}'\underline{D}'^+\underline{c}d\Omega \\ &= \int \underline{D}'\underline{D}'^+d\Omega\underline{c} \\ &= \underline{\underline{I}}\ \underline{c} = \underline{c}\ ; \end{aligned}$$

hence, the noise field N can be expressed as

$$N = (\underline{D}')^+(R') = \underline{D}^+\underline{\underline{Q}}^+\underline{\underline{Q}}\ \underline{R}$$

(12)

$$= \sum_{i=1}^{L} h_i e^{-j\bar{k}\cdot\bar{d}_i}\ ,$$

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where † designates an adjoint operation (changing columns to rows and conjugating the elements) and h_i is a constant resulting from the operation $\underline{D}^\dagger\underline{Q}^\dagger\underline{Q}$. Derivations of equations (6) to (12), which are just the application of standard methods described in most text books,[19],[20] are carried out in great detail here for the purpose of clarifying the notations used and injecting some physical interpretation into the problem.

Transformation of the base functions in equation (8) can be visualized as a way to express $\underline{D}$ and $\underline{R}$ in different coordinates in a multi-dimensional space (Hilbert space). The term "space vector" is purposely avoided for describing the function $\underline{R}$ (underlined in present notation) in order that it will not be confused with the physical space vector, such as $\bar{s}$ and $\bar{k}$ (overbar in present notation) defined previously.

The result given by equation (12) can help to give some understanding about the measurement of noise directionality with an array. If the element spacing of an array were chosen such that h_i is directly related to the cross product of the hydrophone output γ_i, that is,

$$h_i = \gamma_i$$

$$N = \sum_{i=1}^{L} \gamma_i e^{-j\bar{k}\cdot\bar{d}_i} \quad , \tag{13}$$

then the noise field could be deduced immediately.

An example of such a case is a linear array with its hydrophones spaced at half-wavelength intervals (or multiples of a half-wavelength). Even if the element spacings do not form an orthogonal basis, the noise field, according to equation (12), is still a linear combination of the original bases, $\{e^{-j\bar{k}\cdot\bar{d}_i}\}$. Since h_i's are constants, and only $\{\bar{k}\cdot d_i\}$ contains angular information, the angular resolution of the noise field is determined by the array-element spacing. The finest angular resolution of the noise field expressed by equations (12) or (13) that can be achieved will be related to $e^{-j\bar{k}\cdot\bar{d}_L}$, where $|\bar{d}_L|$ is the largest array-element spacing (aperture size).

For broadside response, $\bar{k}$ is perpendicular to $\bar{d}_L$. The maximum value in magnitude can be obtained from the real part of $e^{-j\bar{k}\cdot\bar{d}_L}$:

$$\cos[kd_L \cos\theta]_{\theta = 90°} = 1 \quad .$$

The angular deviation, α, that results in a 3 dB decrease in response can be derived from the following:

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$$\cos[kd_L \cos(90 - \alpha)] = \frac{1}{2}$$

$$kd_L \sin \alpha = \frac{\pi}{3}$$

$$\sin \alpha = \frac{\pi}{3}(\frac{\lambda}{2\pi})\frac{1}{d_L} = \frac{1}{6}(\frac{\lambda}{d_L})$$

$$\alpha \simeq \frac{1}{6}(\frac{\lambda}{d_L}) \ \ (\text{if } \alpha < \frac{1}{2}) \ .$$

(14)

Similarly, for endfire response when $\bar{k}$ is parallel to $\bar{d}_L$, the maximum magnitude of the real part of $e^{-j\bar{k}\cdot\bar{d}_L}$ occurs when

$$\cos[kd_L(\cos \theta - 1)]_{\theta = 0} = 1 \ .$$

The angular deviation β for a 3 dB change in response can be traced from

$$\cos[kd_L(1 - \cos \beta)] = \frac{1}{2}$$

$$kd_L(1 - \cos \beta) = \frac{\pi}{3}$$

$$\cos \beta = 1 - \frac{1}{6}(\frac{\lambda}{d_L})$$

$$\beta \simeq (\frac{\lambda}{3d_L})^{\frac{1}{2}} \ (\text{if } \beta < \frac{1}{2}) \ .$$

(15)

Whether a measurement from an array will attain such an angular resolution for a noise field depends on the actual distribution of the noise field and the uncorrelated noise in the measurement environment. However, the resolution criteria suggested by Bracewell and Roberts,[3,4] $\frac{\lambda}{2d_L}$ in the broadside direction and $\frac{\lambda}{d_L}$ in the endfire direction, are not quite correct. They apply only to the case where each hydrophone element has equal weighting (shading) factor. Bases other than those shown in equation (13) may be used to improve the angular resolution of the processed noise field. The extension and discussions will be presented later.

Finally, it should also be realized from equation (12) that if N is formed by the bases $\{e^{-j\bar{k}\cdot\bar{d}_i}\}$, its coefficients h_i can also be determined

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by substituting equation (12) into equation (7). Let

$$\underline{H} = \begin{vmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ h_k \\ \cdot \\ \cdot \\ h_L \end{vmatrix} \quad , \quad N = \underline{D}^\dagger \underline{H} \quad , \tag{16}$$

and equation (7) becomes

$$\underline{R} = \underline{D} N d\Omega = \int \underline{D} \, \underline{D}^\dagger \underline{H} d\Omega$$

$$= \int \underline{D} \, \underline{D}^\dagger d\Omega \underline{H} = \underline{\underline{A}} \, \underline{H} \quad ;$$

therefore

$$\underline{H} = \underline{\underline{A}}^{-1} \underline{R}$$

and

$$N = \underline{D}^\dagger \underline{\underline{A}}^{-1} \underline{R} \quad ,$$

where $\underline{\underline{A}}^{-1}$ is the inverse of matrix $\underline{\underline{A}}$, which is defined as

$$\underline{\underline{A}} = \int \underline{D} \cdot \underline{D}^\dagger d\Omega$$

$$= \int \begin{vmatrix} 1 & e^{j\bar{k}\cdot(\bar{d}_1 - \bar{d}_2)} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ e^{-j\bar{k}(\bar{d}_1 - \bar{d}_2)} & 1 & \cdot & & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & e^{j\bar{k}\cdot(\bar{d}_k - \bar{d}_{k'})} & \cdot \\ \cdot & \cdot & \cdot & \cdot & & & & \cdot \\ \cdot & \cdot & \cdot & \cdot & e^{-j\bar{k}\cdot(\bar{d}_k - \bar{d}_{k'})} & \cdot & 1 & \end{vmatrix} d\Omega \quad . \tag{18}$$

Nuttall[8] obtained the same results by applying least-square estimation; that is, let

$$\hat{N} = \sum_{i=1}^{L} c_i f_i = \underline{F}^{\dagger} \underline{c} \tag{19}$$

and the integrated square of the error in comparing with the true noise is

$$E = \int (N - \hat{N})^2 d\Omega$$

$$= \int (N - \sum_{i=1}^{L} c_i f_i)^2 d\Omega . \tag{20}$$

By setting $\frac{dE}{dc_i} = 0$ to minimize the error, it was found that

$$f_i = e^{-j\bar{k}\cdot\bar{d}_i} ,$$

and

$$\underline{C} = \underline{\underline{A}}^{-1} \underline{R} . \tag{21}$$

Equations (19) and (21) then agree with equation (17). According to Bessel's inequality,[20]

$$\int (N - \sum h_i e^{-j\bar{k}\cdot\ddot{d}_i})^2 d\Omega \geq 0 ,$$

or

$$\int (N - \underline{D}^{\dagger}\underline{H})^2 d\Omega \geq 0 . \tag{22}$$

The integrated squared error will vanish only when $\underline{H}$ is a complete basis set; that is, a complete basis set for $\underline{H}$ is chosen to represent N. Elaboration on the physical interpretations of this conclusion will be discussed further in the next section.

Array Beamforming Output

The beamforming output of a receiving array has been used as a traditional method to measure the noise field by steering the main lobe of the array, either mechanically or electrically, in various directions. The response of the array is the square of the sum of each hydrophone

output, which can be expressed as:

$$b_k = \int N \left| \sum_{i=1}^{M} W_{ik} e^{j(\bar{k} \cdot \bar{s}_i - ks_i \Omega_k)} \right|^2 d\Omega \ , \qquad (23)$$

where b_k is the array beamforming output for the array steered at Ω_k $\{W_{ik}\}$ are the shading factors applied to the respective array elements. From the results of several independent measurements, b_J, at different steering angles, the vector form of equation (23) can be rewritten as:

$$\underline{B} = \int N \underline{G} d\Omega \ , \qquad (24)$$

where

$$\underline{B} = \begin{vmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ b_k \\ \cdot \\ \cdot \\ b_J \end{vmatrix} \qquad \underline{G} = \begin{vmatrix} \left[\sum_{i=1} W_{ii} e^{j(\bar{k} \cdot \bar{s}_i - ks_i \Omega_1)} \right]^2 \\ \left[\sum_{i=1} W_{ik} e^{j(\bar{k} \cdot \bar{s}_i - ks_i \Omega_k)} \right]^2 \\ \cdot \\ \left[\sum_{i=1} W_{iJ} e^{j(\bar{k} \cdot \bar{s}_i - ks_i \Omega_J)} \right]^2 \end{vmatrix} \qquad (24A)$$

Notice that equation (24) has the same form as equation (7). Hence, the same method used to solve for N in equation (7) (discussed in the last section) can also be applied to equation (24) to deduce N in terms of the measured beamformer output $\underline{B}$ and the array beam pattern $\underline{G}$ at the respective steering angles. The final noise field, N, will be expressed by the bases of $\underline{G}$, so the angular resolution of the computed noise field will depend on the beam pattern of the array. Axelrod, et al.,[3] derived the same result by a least-square criteria approach and described in great detail its application to real-field data processing.

Examining equation (24A) indicates that the bases of $\underline{G}$ are a function of $e^{j\bar{k} \cdot \bar{s}_i}$, which is related to the array-element spacings. Therefore, further simplification of the equation can be pursued to find the direct relationship between the noise field and the array element spacings. Letting

$$\underline{W}_k = \begin{vmatrix} W_{ik}e^{jks_1\Omega_k} \\ \\ W_{2k}e^{jks_2\Omega_k} \\ \cdot \\ \cdot \\ \cdot \\ W_{ik}e^{jks_i\Omega_k} \\ \cdot \\ \cdot \\ \cdot \\ W_{Jk}e^{jks_j\Omega_k} \end{vmatrix} \qquad \underline{E} = \begin{vmatrix} e^{j\bar{k}\cdot\bar{s}_1} \\ \\ e^{j\bar{k}\cdot\bar{s}_2} \\ \cdot \\ \cdot \\ e^{j\bar{k}\cdot\bar{s}_i} \\ \cdot \\ \cdot \\ \cdot \\ e^{j\bar{k}\cdot\bar{s}_J} \end{vmatrix} \quad , \qquad (25)$$

equation (23) can be rewritten as

$$b_k = \int N \; \left| \underline{W}_k^{\dagger}\underline{E} \right|^2 \; d\Omega$$

$$= \int N \; (\underline{W}_k^{\dagger}\underline{E} \; \underline{E}^{\dagger}\underline{W}_k) \; d\Omega \; . \qquad (26)$$

As the values of $\{W_k\}$ are determined by the array spacing, steering angles, and shading factor, they can be factored outside the integral sign, and equation (26) becomes

$$b_k = \underline{W}_k^{\dagger} \int N(\underline{E} \; \underline{E}^{\dagger})d\Omega\underline{W}_k$$

$$= \underline{W}_k^{\dagger} \int N\underline{\underline{D}}d\Omega\underline{W}_k$$

$$= \underline{W}_k^{\dagger}\underline{\underline{R}} \; \underline{W}_k \qquad (27)$$

$$= \sum_{i=1} c_i\gamma_i \; ,$$

where the definitions of $\underline{R}$, $\underline{D}$, γ_i from equation (6) and (7) are used.

Equation (27) gives the relationship between array beamformer output $\underline{B}$ and array hydrophone cross-spectral matrix $\underline{R}$. Since, for a given array with M hydrophones, the maximum independent set of $\underline{R}$ is $L = 1 + \frac{M(M-1)}{2}$, the maximum number of independent measurements from the array beamformer

output for the same array will certainly not exceed L. In the case where the number of measurements obtained from the array beamformer output equals the number of independent $\underline{R}$, according to equations (27),

(7), and (12), the noise field N deduced on the same bases, $\{e^{-j\bar{k}\cdot\bar{d}_i}\}$, should be the same regardless of the shading factor W_k used during the beamforming process.

Such a conclusion can be deduced easily from the physics of the noise field, which will not be changed just because certain shading factors are used in the array beamformer. However, when the number of measurements from the beamformer outputs is less than L, equation (24) must be used to solve for the noise field. Since, under this condition, not all the information received at the input of the array hydrophone is utilized, the noise field deduced via equation (24) will be less accurate in general than that given by equation (7).

REPRESENTATION OF NOISE FIELD

Coherent Noise Sources

The description of a noise field by its directionality is based on the concept that the array is placed in an infinite space (or half-space) and that all the noise sources are in the farfield at a distance further away than L^2/λ (L is the array-aperture size, λ the wavelength). Such an assumption is not always proper for ambient-sea-noise measurements in the real-ocean environment. First, propagation of an acoustic wave in the ocean, particularly over long distances, is influenced by the boundaries of sea surface and ocean bottom, in addition to the sound-velocity profile variation along the propagation path. Even for a single noise source, the wavefront may not be plane and many paths may be taken to reach the measurement site. Second, because of the many interference paths taken by an acoustic wave in the ocean, the noise field (which the array intends to measure) is not only nonstationary but also inhomogeneous. Finally, the problem with nearfield noise sources is that, due to lower propagation loss, they are strongly coherent but do not have plane wave fronts. Those sources will not be described properly, based on the array response from an ambient-noise-directionality measurement. Further interpretation and representation of a noise field in this type of situation is needed.

The conclusion from the analysis in the last section indicates that the basic information about a noise field measured with an array is contained in the input to the hydrophone and the correlation of the hydrophone responses. Such information can be expressed completely for an array with M hydrophones as

$$
\begin{vmatrix}
\gamma_{11} & \gamma_{12} & \cdot & \cdot & \cdot & \gamma_{1M} \\
\gamma_{21} & \gamma_{22} & & & & \cdot \\
\cdot & & & & & \cdot \\
\cdot & & & & & \cdot \\
\cdot & & & & & \cdot \\
\gamma_{M1} & \cdot & \cdot & \cdot & \cdot & \gamma_{MM}
\end{vmatrix}
= \int N
\begin{vmatrix}
1 & e^{j\bar{k}\cdot(\bar{s}_1-\bar{s}_2)} & \cdot & \cdot & e^{j\bar{k}(\bar{s}_1-\bar{s}_M)} \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\cdot & & & & \cdot \\
\cdot & \cdot & \cdot & \cdot & 1
\end{vmatrix}
d\Omega .
\tag{28}
$$

Here the omnidirectional noise levels at each hydrophone, diagonal terms $\{\gamma_{kk}\}$, are not assumed to have the same intensity. Otherwise equation (28) can be reduced to equation (7), which concerns only a homogeneous noise field. Using matrix notation, equation (28) can be rewritten as:

$$
\underline{\underline{P}} = \int N \, \underline{E} \, \underline{E}^{\dagger} d\Omega , \tag{29}
$$

where $\underline{E}$ has already been defined in equation (25).

Expressing the measurement results in the form of equation (25) is based mainly on choosing the particular bases, $\left\{ e^{j\bar{k}\cdot(\bar{s}_\ell - \bar{s}_m)} \right\}$, which may not be the best representation of the noise field that the array is measuring. To examine whether further relationships between $\{\gamma_{\ell m}\}$, which may be affected by the multipath of noise signals, and focusing or diffusing of the noise field, a mathematical technique of diagonalizing a matrix is employed to simplify equation (28). Let

$$
\underline{\underline{\Lambda}} = \underline{\underline{S}}^{-1} \, \underline{\underline{P}} \, \underline{\underline{S}} =
\begin{vmatrix}
\lambda_1 & 0 & \cdot & \cdot & \cdot & 0 \\
0 & \lambda_2 & & & & \cdot \\
\cdot & & \cdot & & & \cdot \\
\cdot & & & \cdot & & \cdot \\
\cdot & & & & \cdot & \cdot \\
0 & \cdot & \cdot & \cdot & \cdot & \lambda_M
\end{vmatrix}
\tag{30}
$$

$$
= \int N (\underline{\underline{S}}^{-1} \underline{E}) \, (\underline{E}^{\dagger}\underline{\underline{S}}) \, d\Omega ,
$$

where Λ is a diagonalized matrix with $\{\lambda_m\}$ as its eigenvalues and $\underline{\underline{S}}$ is a matrix composed of the corresponding eigenvectors.

The physical interpretation of this operation is that if a new measurement basis $(S^{-1}E)$ $(E^{\dagger}S)$ is chosen, the intensity of principal coherent noise sources, $\{\lambda_M\}$, can be determined. In the concept of a Hilbert space, it can be visualized that a set of coordinates is chosen properly in a multidimensional space so that the space vectors, such as P, can be represented simply. According to equation (30), in practice, the coherent noise source can be measured directly by applying its corresponding shading from the eigenvectors of S to the array hydrophone-spacing matrix E. It should be realized that the ability of an array to measure such independent coherent sources depends on the total number of array elements, M. Although the derivation presented here is intended to explore different representations of the noise field, this technique can serve a very practical purpose in signal detection in an ocean environment.

Slowly Varying Noise Field

The angular resolution of the deduced noise field from array measurements is limited by the array-aperture size, as concluded from the previous analysis. This is because all the available information is based on the array-measurement result. However, when a noise field has low variability in its level (very weak additive independent noise) and there is little disturbance to cause uncertainty in the measurement, the noise field can be expressed in terms of other bases by applying analytic continuity to equation (7). For example, delta functions can be employed as bases to represent a noise field attributed to discrete sources:

$$N = \sum_i b_i \delta(\Omega - \Omega_i) \ . \tag{31}$$

Substituting equation (31) into equation (7),

$$\underline{R} = \underline{\underline{\Delta}} \cdot \underline{B} \tag{32}$$

where

$$\underline{B} = \begin{vmatrix} b_1 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_L \end{vmatrix} \qquad \underline{\underline{\Delta}} = \int \begin{vmatrix} e^{j\bar{k}\cdot\bar{d}_1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ e^{j\bar{k}\cdot d_L} \end{vmatrix} \begin{vmatrix} \delta(\Omega - \Omega_1) \cdot \cdot \cdot \delta(\Omega - \Omega_L) \end{vmatrix} d\Omega \ . \tag{33}$$

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Therefore $\{b_\ell\}$ can be solved from the simultaneous equation in (32), and the angular resolution of the noise field is not limited by the array-aperture size. However, depending on the narrowness expected in angular resolution, accurate measurements will be required in $\underline{R}$, which makes equation (32) very sensitive to numerical error (ill-conditioning of equation (32)). This will impose a limitation on the practical application of equations (31) and (32). If there is prior knowledge about the noise field (unfortunately, then there would be less motivation for making a measurement), perhaps only a certain angular section of the noise field would require fine angular resolution for examination. This can be accomplished easily by using non-uniform angular resolution in the expansion of the noise field:

$$N = \sum b_i V(\Omega_i \pm \epsilon_i) \quad , \tag{35}$$

where V is a window function defined by step functions u as

$$V(\Omega_i \pm \epsilon_i) = u(\Omega - \Omega_i + \epsilon_i) - u(\Omega - \Omega_i - \epsilon_i) \quad . \tag{36}$$

Directionality Ambiguity

When the shortest spacing among array hydrophones, $\bar{d}_s$, is larger than a half-wavelength, the deduced noise field, because it contains a term $e^{j\bar{k}\cdot\bar{d}_s}$ according to equation (12), will have at least two peaks in the angular range of Ω. This is the directional ambiguity caused by using such types of arrays in ambient-noise-field measurements. Figure 2 illustrates this problem. The cross spectrum (real part) of two hydrophones plotted versus their space separation has a periodicity of λ in response to a wave coming from the endfire direction. For an array with element separations larger than $\frac{\lambda}{2}$, the sample point "a" in figure 2 will not extract all the information (sample rate is less than Nyquist rate). However, if the general directionality of the noise field is known, an additional data point can be interpolated, such as "b" or "c" in figure 2, for correlation distances less than $\frac{\lambda}{2}$, thus avoiding the directional ambiguity.
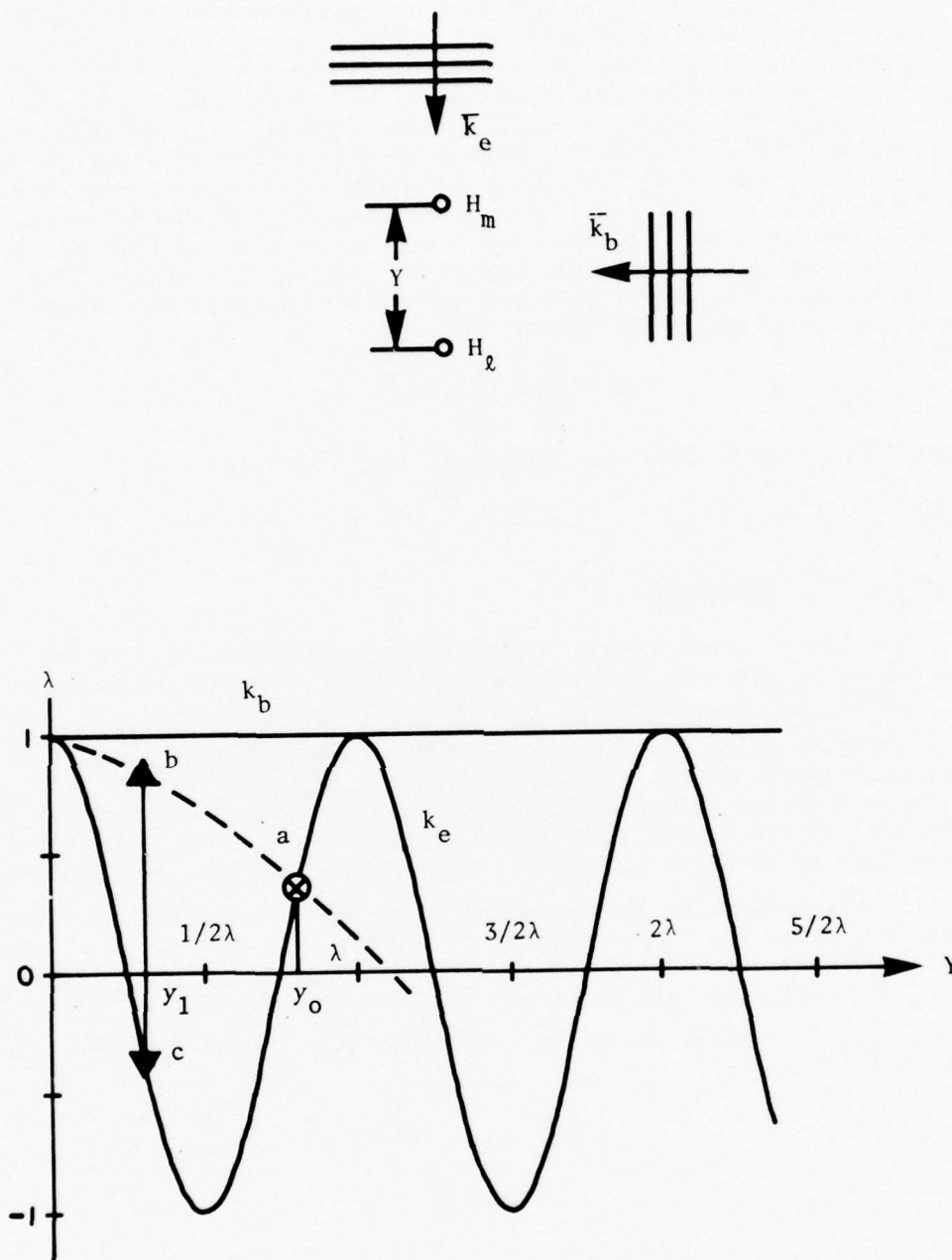
17

Figure 2. Cross Spectrum of Hydrophone Pair in
Response to Directional Wave as a Function
of Their Separation Distance

PROCESSING FOR NOISE DIRECTIONALITY

DATA COLLECTION

The primary data for processing for the noise directionality are either the cross-spectral matrix of the array outputs or its beamformer output, as has been discussed in the previous section. However, if the stationarity and homogeneity of the noise field has to be analyzed beforehand, the original information is contained in each hydrophone output, with the relative phases being preserved. This requires a simultaneous recording of all hydrophone signals. The fluctuations due to array motion and recording-tape skewing will introduce phase and amplitude errors in the recorded hydrophone signal. Methods to correct these errors should be devised before processing the data. If the beamformer outputs (real positive numbers) or the cross-spectral matrix (complex numbers) are recorded instead of the hydrophone signals, information needed to verify any error in the data will be difficult to trace.

By examining the time series of the hydrophone signals, the stationarity of the noise field can be analyzed, so that proper sampling size and rate can be determined for processing for the noise directionality. As the array occupies a certain physical spacing, the problem with an inhomogeneous noise field is rather bothersome. Figure 3 illustrates an example where measurements obtained for the cross spectrum not only changed with hydrophone separation, but also varied according to location. There are two possible ways to process the data: pick a particular location (line CD) and read all correlation values; or take the average value across all positions (line AB). Whatever data are read, the physical interpretation should be implemented with the final result of noise directionality in mind. For optimum beamforming processing[21] in estimating the noise field, the inhomogeneous noise data is absorbed in the computation, hence the physical connection is also lost.

The relationship between the cross spectrum and the noise field is an operation of linear transformation. Therefore, the results of statistical studies on the cross spectrum can apply also to noise directionality if only the mean value is desired.

For a large densely populated array, the original data base is large, and computation, either by orthogonalization or matrix inversion, will introduce large numerical errors in addition to using considerable computer time. A simplification can be made, if the noise field is nearly stationary. By interpolating data points at locations where the base functions become orthogonal, the noise directionality can be deduced from those interpolated data. If the smallest separation of array
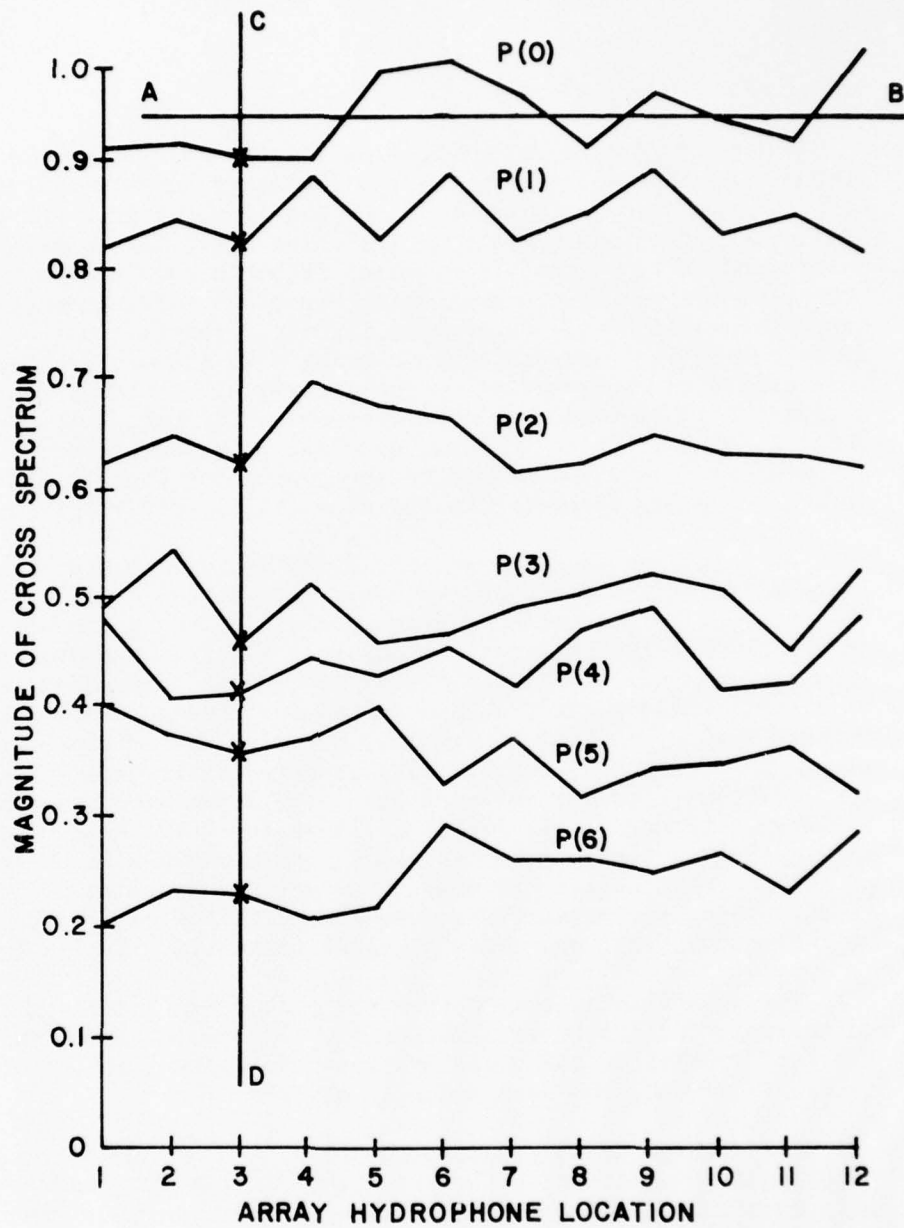
19

Figure 3. Inhomogeneous Noise Field

elements is larger than $\frac{\lambda}{2}$, the interpolating process may eliminate those noise sources whose direction is close to the endfire direction. Such problems should be taken into consideration when the original data are modified.

If the noise field is sure to be stationary and homogeneous, or can be assumed to be so, simultaneous measurement of the cross-spectral matrix is not necessary for deducing the noise directionality. Data from towed arrays, or a long-term average of two hydrophone correlations at various separation distances, can be used directly for processing for noise directionality. Since only a sampling of cross-spectrum data is important to deduce the noise directionality in a homogeneous noise field, an equally-spaced hydrophone array will measure redundant data points. An analytic approach has been devised by Nuttall[22] to position the array elements in such a way that equispatial sampling of the cross-spectral matrix can be achieved.

The angular resolving power of an array can be increased if the certainty of the cross-spectrum data points is improved. Hence, information in analyzing data fluctuation and their statistics will be useful in final processing for the noise directionality.

COMPUTER ALGORITHM

Two alternative mathematical procedures can be used to process the array output for noise directionality: either orthogonalizing the bases, or solving a set of simultaneous equations. There is a standard numerical method for orthonormalization of a set of functions[23] and some available program routines at Naval Underwater Systems Center (NUSC) can be modified easily for this purpose.[24] However, this method involves numerical integration, where accuracy depends on the data segment. Fine angular resolution in noise directionality will take quite a sizable computation and become impractical.

If the original bases of the array are not orthogonal, the other approach for deducing noise directionality, solving a set of simultaneous equations, will be easier, since the matrix element in $\underline{A}$ of equation (18) can be integrated in closed form. A subroutine for double-precision complex-matrix operation is available in NUSC Math Pack.[25] In solving simultaneous equations, the major operation is the matrix inversion, $\underline{\underline{A}}^{-1}$. The accuracy of the numerical computation depends on the matrix size (related to number of array hydrophones) and its rank (the ability of the array to yield independent measurements, related to element spacing). The error introduced during the numerical computation can be estimated from the eigenvalues and determinant of the matrix.[26] When the hydrophones in an array are too close to each other, the matrix $\underline{\underline{A}}$ becomes ill-conditioned, and large errors will exist in the final computation of

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noise directionality. To improve the ill-conditioning situation, but degrade the resolution, a small fraction of deviation can be added in the diagonal terms of matrix $\underset{=}{A}$.

In determining the principal noise sources, resolution of the eigenvalues and the eigenvectors for matrix $\underset{=}{P}$ in equation (30) is needed. A standard computer routine can be found in the NUSC IMSL Program Library. The information contained in $\underset{=}{P}$ can also be used in conventional beamforming and optimum beamforming processing.[21]

There are practical problems in carrying out the numerical computation for finer angular resolution of the noise field, as discussed earlier in this report. First, in order to achieve fine resolution, many samples will be needed from the cross-spectrum function; hence the size of matrix $\underset{=}{\Delta}$ will be very large. Second, because of fixed array aperture, sampling points may be too close to each other, and matrix $\underset{=}{\Delta}$ can easily become ill-conditioned. Meaningful results can be achieved if there are only a few noise sources known beforehand, and thus dense sampling will not be necessary. Therefore, this type of operation applies only to some particular situations.

Because the deduced noise field is based on the summation of certain base functions, if those base functions do not form a sufficiently complete set to express the noise field, the final results of the sum will not be exactly equal to the true noise field. As a result, there will be some oscillation in the deduced noise field data (Gibbs phenomenon).[20] Smoothing techniques, weighting, or iteration have been suggested by Nuttall[8,22] for the processing computations to reduce such phenomenon. However, in practice, the noise field is not known beforehand. Therefore, there is no reference to the method for applying the smoothing technique to improve the estimated noise field (smoothing actually degrades the angular resolution of the estimated noise field).

EXAMPLE

GENERAL

The analysis results in the previous sections apply to any type of array in general. However, the examples illustrated here have been limited to a 240 m vertical array operated at 50 Hz. This is the array developed for AUTOBUOY as a portable system for ambient-sea-noise directionality measurements.[17,18] The array has 12 elements spaced unequally in order to make broad frequency band (30 Hz to 250 Hz) measurements. Outputs from 12 hydrophones can be recorded simultaneously. There are field data available from a recent deployment. Hence, comparisons

can be made between the simulated cases and the real ocean environment in processing for the noise directionality.

Because of the geometry of a vertical array, as shown in figure 4, only the average noise in the cone at various vertical angles, θ, can be measured. Hence, the general relationship of the noise field and the array parameters, such as equation (17), can be reduced to a one-dimensional problem:

$$\underline{R} = \int_0^\pi N(\theta)\underline{D} \sin \theta \, d\theta \ , \tag{37}$$

where

$$N(\theta) = \frac{1}{2\pi} \int_0^{2\pi} N(\theta,\phi) d\phi \ , \tag{38}$$

and all $\{\bar{k}\cdot\bar{d}_i\}$ can simply be replaced by $kd_i \cos \theta$. The mathematical manipulation can be made even simpler by changing the variable $\sin (\frac{\pi}{2} - \theta)$ to u. Equation (37) becomes

$$\underline{R} = \int_{-1}^1 N(u)\underline{D}du \ , \tag{39}$$

and

$$kd_i \cos \theta \longrightarrow kd_i u \ . \tag{40}$$

NOISE SIMULATION

Figure 5 shows the estimated noise directionality for a broadside (θ = 90 deg) noise deduced from the cross-spectral matrix of 12 hydrophone outputs. It can be observed that there are small oscillations around zero for $\theta \neq 90$ deg. This is the Gibbs phenomenom due to representing the noise field by a finite series of base functions. The cross-spectrum matrix $\underline{P}$ in this case is:
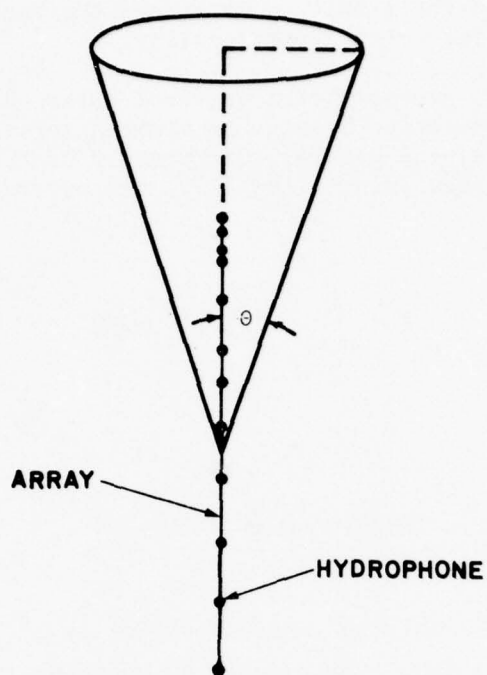
Figure 4.   Geometry of a Vertical Line Array

$$\underline{\underline{P}} = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & & & \text{all elements} = 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{vmatrix} \quad .$$

It has only one nonzero eigenvalue, 12, and the corresponding eigenvector is:

$$\underline{S} = \frac{1}{\sqrt{12}} \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{vmatrix} \quad . \tag{41}$$
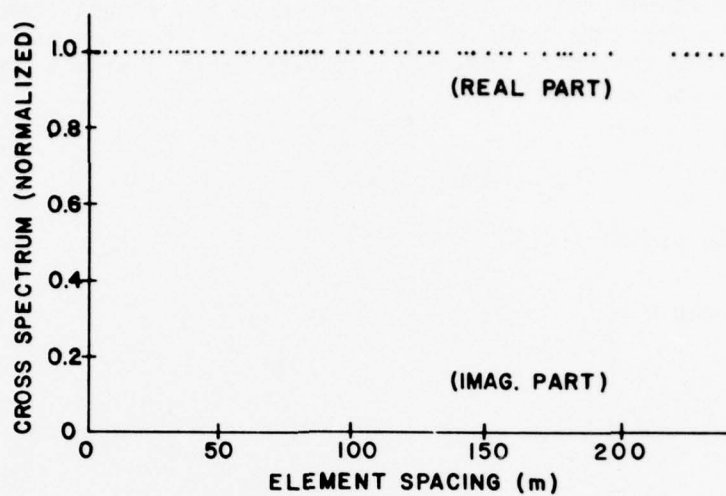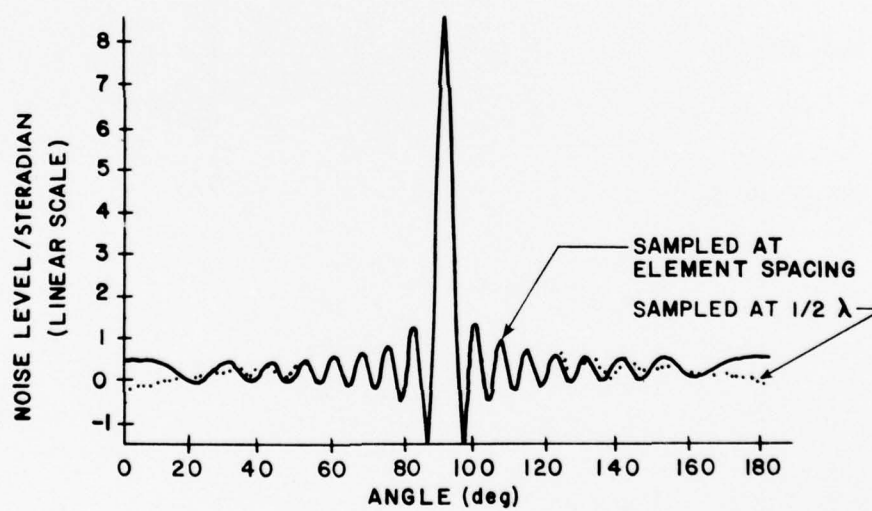
24

Figure 5a.   Cross Spectrum

Figure 5b.   Estimated Noise Directionality

Figure 5.   Broadside Simulated Noise Field

This set of values can be used as shading on the array; then the array output at a steering angle of 90 degrees will be equal to the noise level. Since the above analysis indicates that there is only one noise source, we can assume

$$N = a\delta(\theta - \theta_0) . \tag{42}$$

According to equation (33)

$$\underline{\underline{\Delta}} = \int_0^\pi \begin{vmatrix} e^{ikd_1 \cos \theta} \\ e^{ikd_2 \cos \theta} \\ \cdot \\ \cdot \\ \cdot \\ e^{ikd_{12} \cos \theta} \end{vmatrix} \begin{vmatrix} \delta(\theta - \theta_0) & 0 & 0 \cdot \cdot 0 \end{vmatrix} \sin \theta \, d\theta \tag{43}$$

$$= \begin{vmatrix} e^{ikd_1 \cos \theta_0} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ & & & & & & \cdot \\ e^{ikd_2 \cos \theta_0} & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ \cdot & & & & & & \cdot \\ e^{ikd_{12} \cos \theta_0} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \end{vmatrix} \sin \theta_0 .$$

$$1 = a \, e^{ikd_1 \cos \theta_0} \sin \theta_0$$

$$1 = a \, e^{ikd_2 \cos \theta_0} \sin \theta_0$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \tag{44}$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$1 = a \, e^{ikd_3 \cos \theta_0} \sin \theta_0 .$$

The only possible solution for equation (44) is

$$a = 1$$
$$\theta_0 = \frac{\pi}{2} ; \tag{45}$$

26

hence

$$N = 16(\theta - \frac{\pi}{2}) \ .$$

This checks with the original assumption of a broadside noise source.

For an isotropic noise field, the cross spectrum can be expressed analytically as

$$\gamma_i = \frac{\sin(kd_i)}{kd_i} + jo \ . \qquad (46)$$

The graphical display of this function and its deduced noise directionality are shown in figure 6. According to equation (46), the cross spectrum γ_i equals zero at multiple half-wavelength spacing. As the 240 m array corresponds to a length of 15.56 half-wavelengths at 50 Hz, the eigenvalues of the cross-spectral matrix in this case will have at most 15 nonzero values. These 15 eigenvalues, represented as 15 independent noise sources, do not completely describe the isotropic noise field. Therefore, the method discussed previously is not applicable.

The cross-spectral matrix due to uncorrelated local noise sources, such as system and turbulence noise generated at each hydrophone element, already has the diagonalized matrix form. Its eigenvalues will not depend on the length of an array and the locations of the array elements. For an array shorter than a half-wavelength, the cross spectrum contributed by isotropic noise field, $\sin(kd)/(kd)$ will be approximately one. Thus the cross-spectral matrix (rank one matrix) will have only one dominant eigenvalue. However, the number of eigenvalues due to uncorrelated local noise sources will not be changed. Therefore, in a super-directivity array application, no advantage can be taken by adjusting the shading of array elements (changing array response in different-representation coordinates, or base functions) to eliminate the interference caused by uncorrelated local noise sources.

The effect of nearfield noise sources on the array response is illustrated by a simulated case, shown in figure 7. As the array aperture is 240 m, for a 50 Hz signal, the criterion for farfield is

$$\frac{a^2}{\lambda} = 2020 \text{ m} \ .$$

The geometry depicted in figure 7 gives the relationship to determine the cross spectrum among the hydrophones. Figure 8 shows the estimated noise-field directionality based on the cross spectrum computed
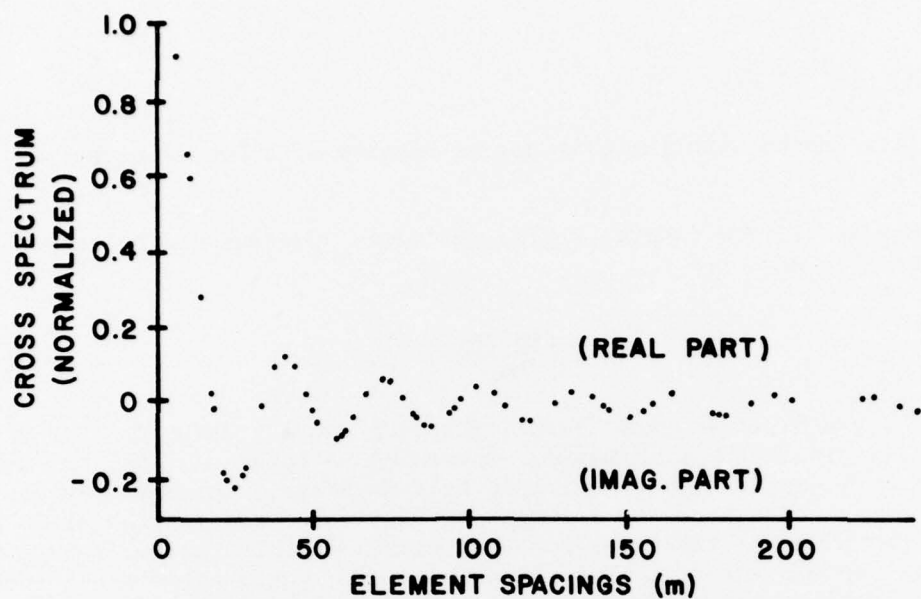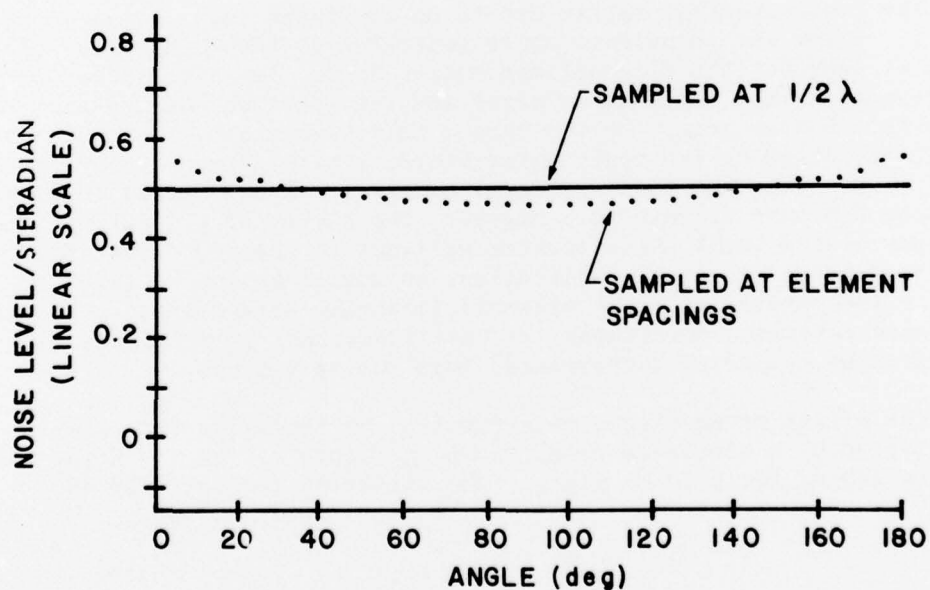
Figure 6a.   Cross Spectrum

Figure 6b.   Estimated Noise Directionality
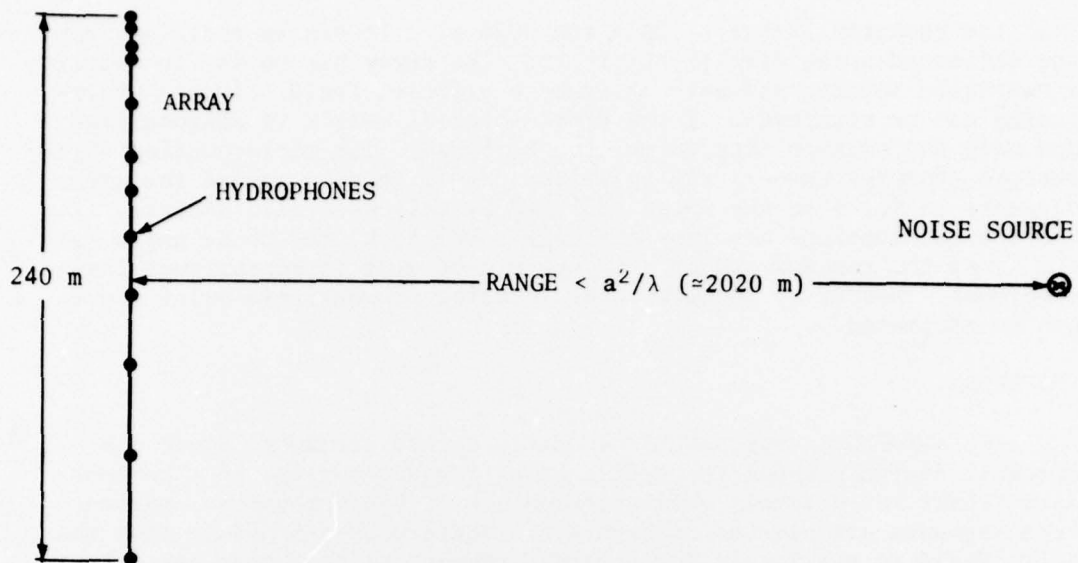
Figure 6.   Isotropic Noise Field
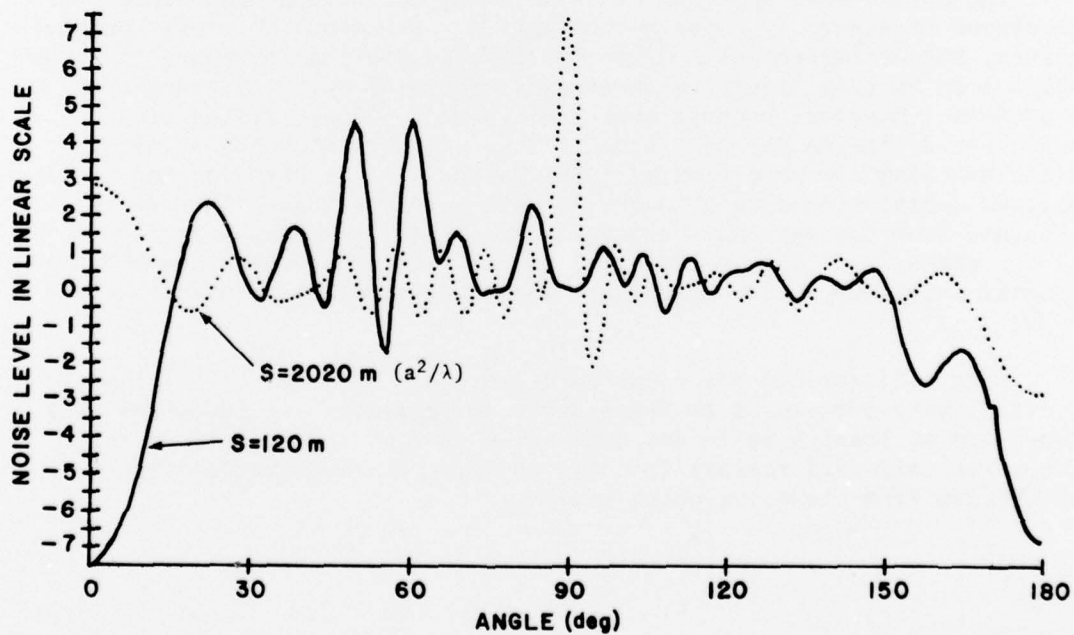
Figure 7. Noise Source in Nearfield

Figure 8. Estimated Noise Field Due to Nearfield Noise Source

from the geometry (range = 120 m and 2020 m). It can be observed from the estimated noise directionality that the array has no way to detect a nearfield source, and only observes a diffused field. Such a short-coming can be eliminated if the cross-spectral matrix is diagonalized and only one nonzero eigenvalue, 12, is found. The corresponding eigen-vectors (complex number) will give the proper shading needed for array elements to focus on the noise radiated by this nearfield source. If the element spacings are less than one wavelength, the phase angle (argument of the complex number shading) can be used to reconstruct the wavefront. Hence, by geometry, the location of nearfield noise source can be estimated.

SEA DATA

The AUTOBUOY array has 12 unequally-spaced elements, since the original design planned to cover a broad frequency range.[18] The omni-directional noise levels obtained from the 12 hydrophones for various time segments are plotted in figure 9. These curves indicate that the noise field is stationary for a sample segment greater than one minute. However, the variation in the noise level among the hydrophones suggests that the noise field is not quite homogeneous (some variation may also be caused by system noise, such as that caused by bad connections).

The noise cross spectrum at 50 Hz among the hydrophone outputs is displayed in figure 10 after normalization. Based on the cross-spectral curves, the estimated noise directionality is plotted in figure 11. The solid line in that figure is obtained from the 67 pairs of cross-spectral data. However, because some hydrophone spacings are too close, 5 percent deviation has been added in the diagonal elements of matrix $\underline{\underline{A}}$ for avoiding ill-conditioning. The dashed line is based on the inter-polated correlation data at multiple half-wave spacings. The result obtained from the optimum beamforming process[18] is included in figure 12 for comparison (only in relative scale). It can be observed that the general intensity distributions are about the same, particularly at the peaks.

Figure 13 depicts the computed eigenvalues from the noise cross-spectral matrix measured at different time segments. It indicates that there are at least 9 major dominant noise sources within a 10 dB range. The curve falls off rapidly from #10 to #12. The background noise is 30 dB down from the major noise sources.
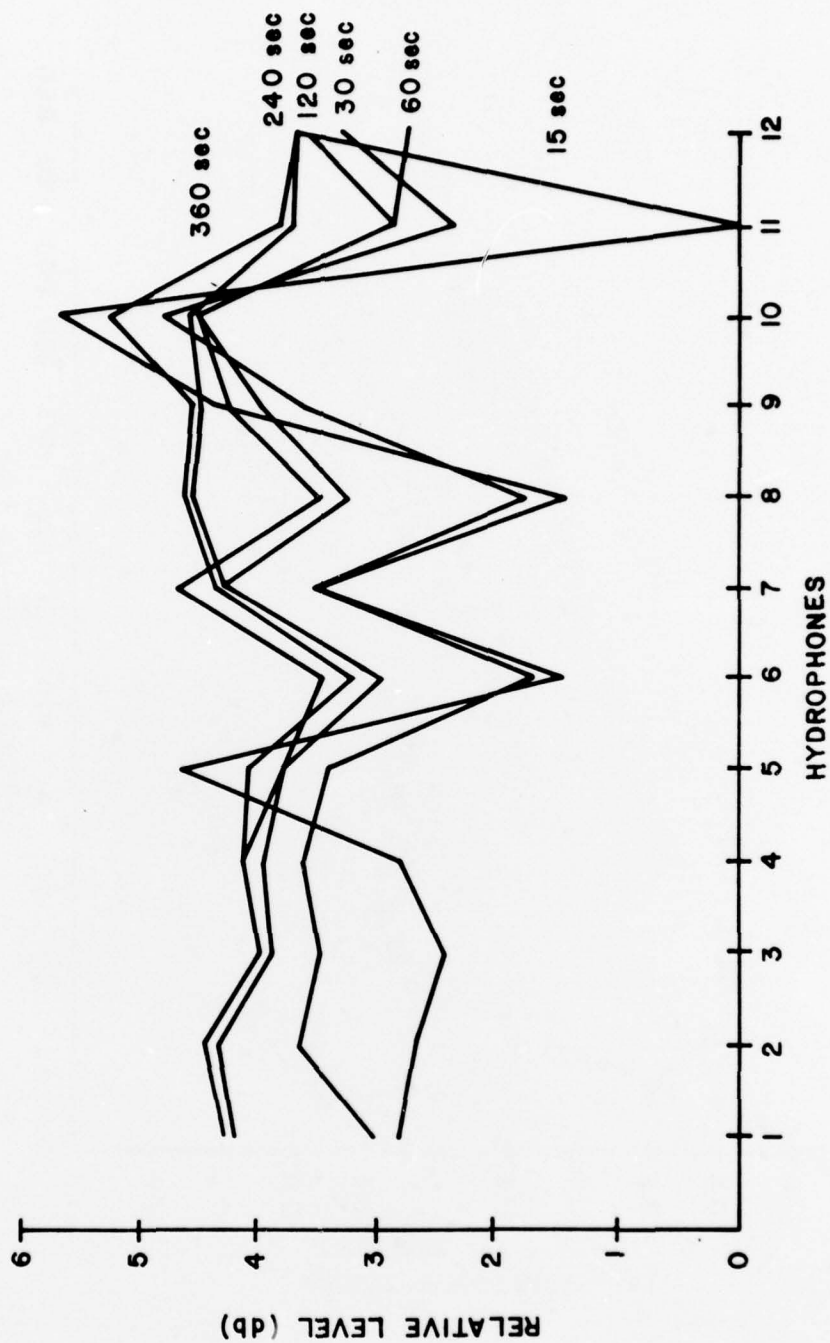
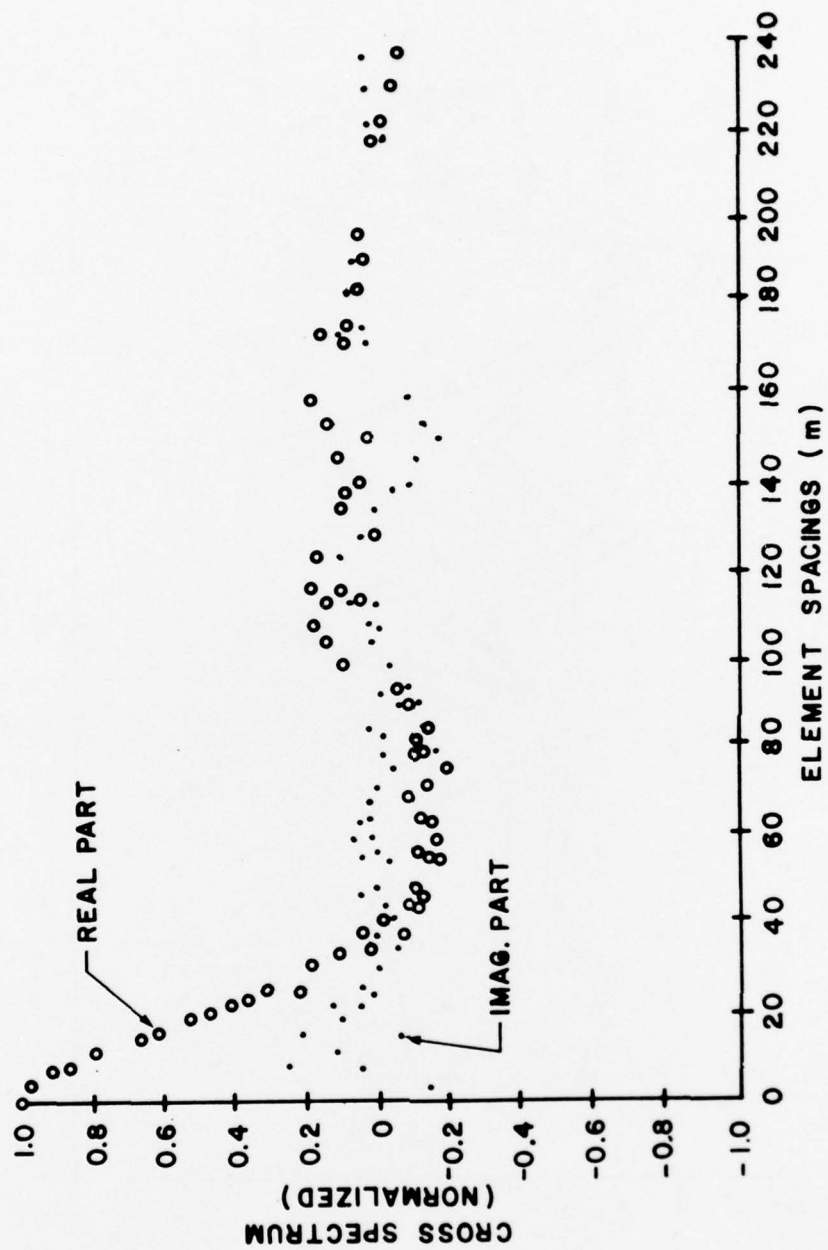Figure 9. Stationarity and Homogeneity of Noise Field

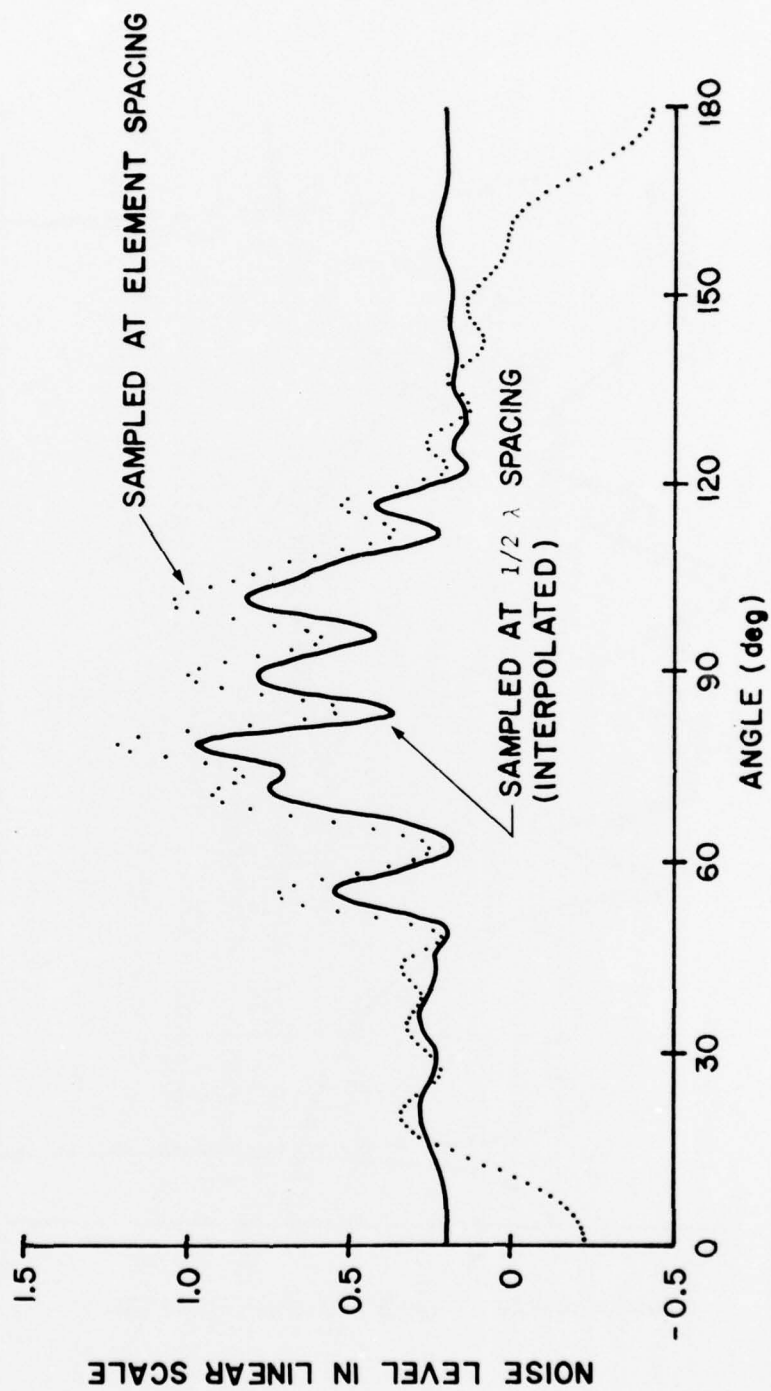Figure 10. Cross Spectrum of a Noise Field
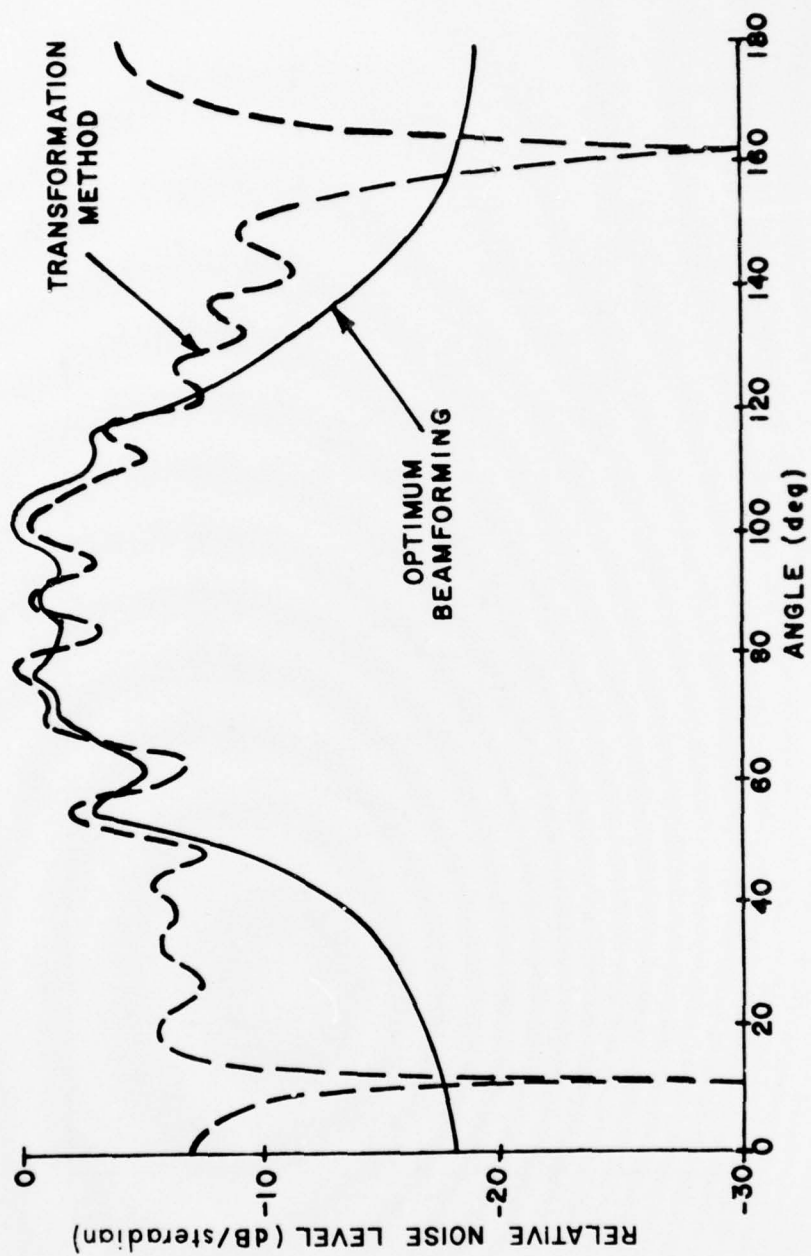
Figure 11. Estimated Noise Field

Figure 12. Comparison of Optimum Beamforming and Noise
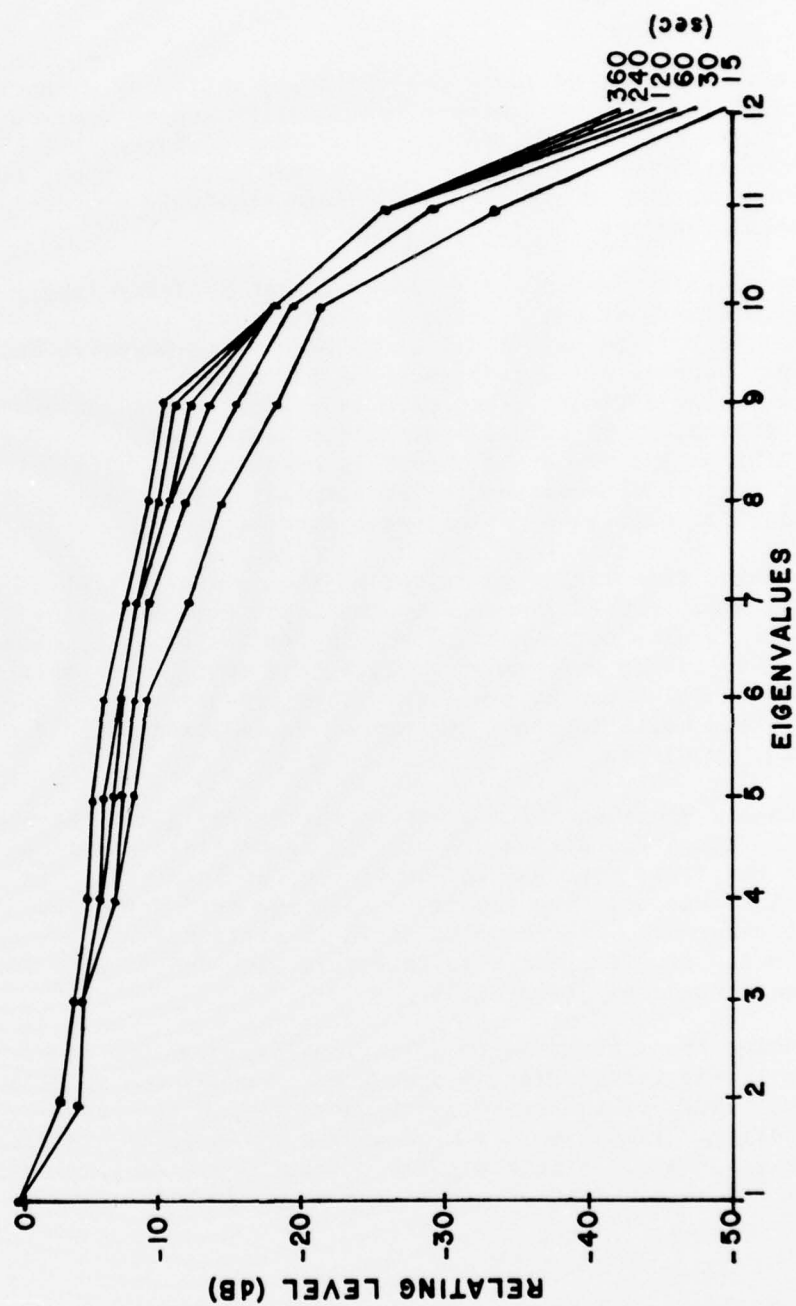Field Estimation by Transformation Method

Figure 13. Eigenvalues of a Noise Field for a 12-Element Line Array

CONCLUSIONS

From an analysis of the noise field and the array parameters, it is indicated that ambient-noise directionality can be derived directly from the basic information contained in array measurements, the cross spectrum of hydrophone outputs. If the same array is used, other types of measurements should also reach the same result if processing methods are correctly applied.

Theoretically, the noise field which can be determined from the array measurements is not limited by the aperture size of the array if the noise field is composed only of plane waves. However, in practical situations, because of interference due to uncorrelated and partially-correlated noise sources, along with numerical errors introduced during the computation, it is not easy to achieve such a result. The most reliable solution (principal solution) is based on the array structure. For a line array, computation can be simplified if cross-spectral data are obtained at multiples of half-wave spacings.

The basic information contained in the cross spectrum of an array output can also be used to evaluate the major coherent noise sources (eigenvalues). The corresponding shading factor on array elements (eigenvectors) also can be determined. Hence, the optimum response of an array to such noise can be achieved. This process is very useful to detect nearfield noise sources, but cannot be implemented in the noise-directionality display.

Since the cross spectrum of the array output gives the best description of the noise field, measurements of noise field should be designed to sample the cross spectrum at equally-spaced intervals. If the noise field is homogeneous, then the array spacings need not necessarily be equal. Furthermore, if the noise field is stationary, cross-spectral data collected at different time intervals also can be used for processing for ambient-noise directionality.

Deducing the ambient-noise directionality from the cross-spectral data is a linear transformation operation. Hence, the results of the statistical study of the cross spectrum can apply also to ambient-noise directionality. If there is a high degree of certainty in cross-spectral data (low fluctuation), fine angular resolution of ambient-noise directionality can be achieved by proper use of prior knowledge.

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REFERENCES

1. B. A. Becker, The Directional Distribution of Ambient Noise in the Ocean, Scripps Institute of Oceanography, ref. 61-4, 7 March 1961.

2. G. R. Fox, "Vertical Directionality of Ambient Noise in the Ocean," Journal of the Acoustical Society of America, vol. 35, p. 811, 1963.

3. E. H. Axelrod, B. A. Schoomer, and W. A. Von Winkle, "Vertical Directionality of Ambient Noise in the Deep Ocean at a Site near Bermuda," Journal of the Acoustical Society of America, vol. 37, pp. 77-83, 1965.

4. R. N. Bracewell and J. R. Roberts, "Aerial Smoothing in Radio Astronomy," Australian Journal of Physics, vol. 7, pp. 615-640, 1954.

5. Y. T. Lo, "On the Theoretical Limitation of a Radio Telescope in Determining Sky Temperature Distribution," Journal of Applied Physics, vol. 32, pp. 2052-2054, 1961.

6. M. W. Anderson and R. L. Tittle, "Analysis of an Integral Equation Arising from the Investigation of Vertical Directivity of Sea Noise," Journal of the Acoustical Society of America, vol. 45, pp. 1129-1133, 1969.

7. R. N. McDonough, "Spectrum Restoration for Uniformly Spaced Arrays," Journal of the Acoustical Society of America, vol. 57, pp. 913-922, 1975.

8. A. H. Nuttall, "Estimation of Noise Directionality Spectrum," NUSC Technical Memorandum No. TC-211-71, 20 October 1971. Also NUSC Technical Report 4345, 1 September 1972.

9. R. N. McDonough, "Deconvolution Processing for a Nonuniform Array," Journal of the Acoustical Society of America, vol. 59, pp. 1406-1411, 1976.

10. D. Lee, Maximization of Reverberation Gain, NUSC Technical Report 5375, 2 October 1976.

11. J. Faran, Jr., and R. Hills, Jr., "Wideband Directivity of Receiving Array," Journal of the Acoustical Society of America, vol. 57, pp. 1300-1308, 1975.

12. R. L. Streit, "Optimal Shading of the General Line Array," Journal of the Acoustical Society of America, vol. 58, p. S9 (E5), 1975.

13. R. L. Streit, "Optimized Symmetric Discrete Line Arrays," IEEE Transactions, Antenna Propagation, Ap-23, pp. 860-862, 1975.

14. A. Z. Robinson, "A New Approach to Superdirective Arrays," NRL Report 7241, 5 January 1971.

15. A. H. Nuttall and D. W. Hyde, A Unified Approach to Optimum and Suboptimum Processing for Arrays, NUSC Technical Report 992, 22 April 1969.

16. N. L. Owsley, personal communication.

17. R. S. Gozzo and R. L. Martin, AUTOBUOY, Present Capabilities and Planned Modifications, NUSC Technical Report 4505, 11 June 1973.

18. R. L. Martin, R. S. Gozzo, and N. Yen, Directional AUTOBUOY Design, Performance Tests, and Acoustic Results, NUSC Technical Report 5491 (in preparation).

19. P. M. Morse and H. Feshback, Methods of Theoretical Physics, McGraw-Hill Book Co., Inc., New York, 1961.

20. R. Courant and D. Hilbert, Methods of Mathematical Physics, Interscience, New York, 1937.

21. N. Yen, "Data Reduction for AUTOBUOY Array Ambient-Noise-Directionality Measurement," NUSC Technical Memorandum No. TA11-11-74, 21 January 1974.

22. A. H. Nuttall, "Estimation of Noise Directionality Spectrum: Extensions and Generalizations," NUSC Technical Memorandum No. TC-6-73, 7 May 1973.

23. E. Isaacson and H. B. Keller, Analysis of Numerical Methods, John Wiley and Sons, Inc., New York, 1966.

24. D. Lee, personal communication.

25. R. Streit, Complex Matrices, Digital Computer Memorandum #61, 23 July 1975, NUSC.

26. A. Ralston, A First Course in Numerical Analysis, McGraw-Hill Book Co., Inc., New York, 1965.

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